Convex Optimization for Machine Learning and Computer Vision

Tutorial

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Contents

Minimizer

Theorem

Any proper function $J : \mathbb{E} \to \overline{\mathbb{R}}$, which is **bounded from below**, **coercive**, and **lsc**, has a (global) minimizer.

- Above theorem works for all proper functions.
- Global minimizer and local minimizer.
- strictly convex ensures uniqueness.

Subdifferential

Subdifferential

The **subdifferential** of a **convex** function $J : \mathbb{E} \to \bar{\mathbb{R}}$ at $u \in \mathrm{dom} J$ is defined by

$$\partial J(u) = \{p \in \mathbb{E} : J(v) \geq J(u) + \langle p, v - u \rangle, \forall v \in \mathbb{E}\}$$

- relation to supporting hyperplane.
- a set-valued map. Monotone operator. convex, nonempty, compact.
- special case.
- calculus(requirements): chain rule, summation rule.
- optimality condition and normal cone.

Convex Conjugate

Convex Conjugate

Given a function $J : \mathbb{E} \to \bar{\mathbb{R}}$, the **convex conjugate** (a.k.a. Legendre-Fenchel transform) of J is defined by

$$J^*(p) = \sup_{u \in \mathbb{E}} \langle u, p \rangle - J(u), \forall p \in \mathbb{E}.$$

- Properties: scalar, translation.
- $J(u) + J^*(p) \geq \langle u, p \rangle$, "=" iff $p \in \partial J(u)$.