

Variational Methods for Computer Vision: Exercise Sheet 3

Exercise: 14 November 2013

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a convex real valued function. A point $\tilde{x} \in \mathbb{R}^n$ is a local minimizer of f if there exists a neighborhood $\mathcal{N}(\tilde{x})$ such that $f(\tilde{x}) \leq f(x) \quad \forall x \in \mathcal{N}(\tilde{x})$. A stationary point of f is a point at which the gradient vanishes hence a point x^* which satisfies the following equation:

$$\nabla f(x^*) = 0$$

Prove the following statements:

- (a) Every local minimizer of f is a global minimizer.
 - (b) Suppose f is additionally differentiable. Every stationary point of f is a global minimizer.
2. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a real valued function. The epigraph of f is the following set:

$$\text{epi } f := \{(u, a) \in \mathbb{R}^n \times \mathbb{R} \mid f(u) \leq a\}$$

Prove that f is convex if and only if its epigraph is a convex set.

3. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and let $g : \mathbb{R}^n \rightarrow \mathbb{R}$ be real valued convex functions. Show whether or not the following functions are convex:

(a)

$$g(x) := \alpha f(x) + \beta g(x) \quad \text{s.t. } \alpha, \beta > 0$$

(b)

$$g(x) := \max(f(x), g(x))$$

(c)

$$g(x) := \min(f(x), g(x))$$

4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be twice differentiable convex functions. Find the condition on f that assures the function:

$$h(x) := f(g(x))$$

is convex by using the fact that function h is convex if and only if $h(x)'' \geq 0$.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Finish the practical exercise from last sheet.
2. In the lecture we encountered the following cost function for denoising images:

$$E_{\lambda}(u) = \frac{1}{2} \sum_{i=1}^N (f_i - u_i)^2 + \frac{\lambda}{2} \sum_{i=1}^N \sum_{j \in \mathcal{N}(i)} (u_i - u_j)^2.$$

where u is the sought image, f is the input image and where $\mathcal{N}(i)$ denotes the neighborhood of pixel i . Minimize the above function by solving the linear system of equations which arises from the optimality condition, using the Gauss-Seidel method. Initialize your solution with 0.

Matlab-Tutorials:

<http://www.math.utah.edu/lab/ms/matlab/matlab.html>

<http://www.math.ufl.edu/help/matlab-tutorial/>

<http://www.glue.umd.edu/~nsw/ench250/matlab.htm>