Variational Methods for Computer Vision: Exercise Sheet 3

Exercise: 18 November 2011

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $L : X \to Y$ be a linear operator and X, Y be finite dimensional spaces with $dim\ X = n$ and $dim\ Y = m$. Let $\{e_1, ..., e_n\}$ and $\{\tilde{e}_1, ..., \tilde{e}_m\}$ be the bases for X and respectively for Y. Show that the operator L can be represented by an $m \times n$ matrix M, hence:

$$L(u) = M \cdot u \quad \forall u \in X$$

2. The general diffusion equation for a function can be written as follows:

$$\partial_t u(x,t) = \operatorname{div}(D(x,t)\nabla u(x,t))$$

where the so called diffusion tensor $D \in \mathbb{R}^{2\times 2}$ is a symmetric positive definite matrix and $u \in C^2(\Omega \times \mathbb{R}_0^+; \mathbb{R})$ with $\Omega \subset \mathbb{R}^2$ describes the complete diffusion process and solves the partial differential equation. The Eigenvectors v_1 and v_2 of D indicate the direction of the diffusivity. The corresponding eigenvalues λ_1 and λ_2 express the diffusivity along these vectors.

Using solely the gradient magnitude as an edge detector in the diffusivity has the disadvantage that oscillations may be misinterpreted as edges which would be preserved in an edge preserving diffusion. A remedy for this problem is using instead the gradient of the convolved function

$$\nabla u_\sigma = \nabla(K_\sigma * u)$$

with a Gaussian kernel K_σ instead of ∇u.

Suppose we choose:

$$v_1 \| \nabla u_\sigma \ , \quad v_2 \perp \nabla u_\sigma$$

hence v_1 is parallel to ∇u_σ and v_2 is perpendicular to ∇u_σ. Write down the diffusion tensor D.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. In the first theoretical exercise we showed that every linear operation on a finite dimensional space can be represented as a matrix vector multiplication. Since the convolution operation is linear its possible to represent it as such. In the last exercise we implemented the separable formulation of the convolution with a Gaussian kernel. Write a script that implements the same procedure as a Matrix vector multiplication.

   Hint: stack the image for this purpose in a vector using the `reshape` command.

2. Download the archive file `vmcv_ex03.zip` from the homepage and unzip it in you home folder. Use the template file `difusion_filter.m` for a non linear diffusion filter and complete the missing code at line 58 if you have not done it last time. Test the script on the image `lena.png`. Now test the same script on the image `triangle.gif`. What do you observe?

3. Instead of using ∇u utilize a smoothed version ∇u_σ where u_σ is the convolved version of u with a Gaussian kernel of standard deviation σ in order to construct the diffusivity. What can you say about the result?

4. Modify the code in `difusion_filter.m` in order to implement an anisotropic version of the nonlinear diffusion using the diffusion tensor discussed in exercise 3 of the theory part of this exercise sheet and set its eigenvalues as follows:

$$\lambda_1 = \frac{1}{1 + |\nabla u_\sigma|^2 / k^2} \qquad \lambda_2 = 1$$

   Where k is a positive contrast parameter.

Matlab-Tutorials:
`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`
`http://www.glue.umd.edu/~nsw/ench250/matlab.htm`