Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 5

Room: 02.09.023

Wed, 21.06.2017, 14:00-16:00

Submission deadline: Tue, 20.06.2017, 23:59 to laehner@in.tum.de

Mathematics: First Fundamental Form

Exercise 1 (2 points)**.** Consider the following coordinate maps from Exercise sheet 3:

$$c_i : [0,1] \to \mathbb{R}^2$$
$$c_1 : t \mapsto \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}, \qquad c_2 : t \mapsto \begin{pmatrix} \cos(t^2) \\ \sin(t^2) \end{pmatrix}$$

Calculate the first fundamental form of both parametrizations.

Exercise 2 (3 points)**.** Consider the coordinate map (from Exercise Sheet 2, $C = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$):

$$x : C \to \mathbb{R}^3$$
$$(u,v) \mapsto (u, v, \sqrt{1 - u^2 - v^2})$$

1. Calculate the first fundamental form for each point on the surface.
2. Integrate the length of the straight line l_1 between $(0,0) \in C$ and $(0,0.8) \in C$ both on C and the manifold.
3. Let l_2 be the line between $(0,0)$ and $(0.565685, 0.565685)$. Calculate the angle between l_1, l_2 on the domain and on the manifold.

Exercise 3 (3 points)**.** Show that the first fundamental form is invariant to rotation and translation in the coordinate map. Let $x_1, x_2 : \mathbb{R}^2 \to \mathbb{R}^3$ be defined such that $x_2(u) = R \cdot x_1(u) + T$ where $R \in \mathbb{R}^{3\times3}$ is a rotation matrix and $T \in \mathbb{R}^3$. To show is that:

$$Dx_1(u)^\top Dx_1(u) = Dx_2(u)^\top Dx_2$$