Analysis of Three-Dimensional Shapes
F. R. Schmidt, M. Vestner, Z. Lähner
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Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 4
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Mathematics: Linear Algebra Recap

Let X be a vector space. An *inner product* is a function $f : X \times X \to \mathbb{C}$ with the following properties:

1. $f(x, x) \geq 0 \quad \forall\ x \in X$ and $f(x, x) = 0 \Leftrightarrow x = 0$

2. $f(x, y) = \overline{f(y, x)}$

3. $f(x + \alpha x', y) = f(x, y) + \alpha f(x', y) \quad \forall\ x, x', y \in X, \alpha \in \mathbb{C}$

The standard inner product is defined as $\langle x, y \rangle = x^\top \bar{y},\ x, y \in \mathbb{R}^n$. If $M \in \mathbb{R}^{n \times n}$ is a symmetric, positive definite matrix an M-inner product can be defined by taking $\langle x, y \rangle_M = x^\top M y$.

A linear operator $T : X \to X$ is called *self-adjoint* w.r.t. an inner product if the following holds:
$$\langle Tx, y \rangle = \langle x, Ty \rangle$$

An *eigenvector* is an element $0 \neq x \in X$ for which there exists a scalar $\lambda \in \mathbb{C}$ such that
$$Tx = \lambda x$$

The scalar λ is called *eigenvalue*.

Exercise 1. Let $L = M^{-1}S \in \mathbb{R}^{n \times n}$ be self-adjoint w.r.t. the M inner product. Show that the following statements hold.

1. S is symmetric (self-adjoint) w.r.t. to the standard inner product.

2. The eigenvalues of L are real.

3. The eigenvectors v_i, v_j with respective eigenvalues $\lambda_i \neq \lambda_j$ are orthogonal.

4. $v_1, ..., v_k$ are eigenvectors of L with the same eigenvalue λ, then $\sum_i \alpha_i v_i$ is also an eigenvector with eigenvalue λ.