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Machine Learning for Robotics and Computer Vision
Summer term 2016

Homework Assignment 1
Topic 1: Linear Algebra
April 20, 2016

Exercise 1: Warm up

a) What multiple of $a = (1, 1, 1)$ is closest to the point $b = (2, 4, 4)$? Find also the closest point to a on the line through b.

b) Prove that the trace of $P = aa^T/a^Ta$ always equals 1.

c) Show that the length of Ax equals the length of A^Tx if $AA^T = A^TA$.

d) Which 2×2 matrix projects the x,y plane onto the line $x + y = 0$?

Exercise 2: Determinants

a) If a square matrix has determinant $\frac{1}{2}$, find $\det(2A)$, $\det(-A)$, $\det(A^2)$ and $\det(A^{-1})$.

b) Find the determinants of

$$A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \quad , \quad U = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}, U^T \text{ and } U^{-1}$$

Exercise 3: Eigenvalues and Eigenvectors

a) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

, their traces and their determinants.

b) Using the characteristic polynomial, find the relationship between the trace, the determinants and the eigenvalues of any square matrix A.

c) Diagonalize the unitary matrix V to reach $V = U\Lambda U^*$. All $|\lambda| = 1$. $V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$

d) Suppose T is a 3×3 upper triangular matrix with entries t_{ij}. Compare the entries of T^*T and TT^*. Show that if they are equal, then T must be diagonal. (All normal triangular matrices are diagonal)

Exercise 4: Singular Value Decomposition

a) Find the singular values and singular vectors of

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$$

b) Explain how $U\Sigma V^T$ expresses A as a sum of r rank-1 matrices: $A = \sigma_1 u_1 v_1^T + \ldots + \sigma_r u_r v_r^T$

c) If A changes to $4A$ what is the change in the SVD?

What is the SVD for A^T and for A^{-1} ?

d) Find the SVD and the pseudoinverse of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

and $C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Topic 2: Probabilistic Reasoning

Exercise 5: Follow the robot

Assume you get your hands on a robot that can measure its distance to a wall in front of it. You model this using a continuous random variable with a Normal distribution $\mathcal{N}(x; \mu, \sigma^2)$.

a) The robot also has a camera on board that is not color-calibrated correctly so the color mapping is probabilistic and looks like the following table:

z \ x	**R**	**G**	**B**
R	0.8	0.1	0.1
G	0.1	0.6	0.2
B	0.1	0.3	0.7

For instance, the probability that the robot reads blue while the true color is green is $p(z = B|x = G) = 0.3$
Assume the robot is located in a white room with 5 boxes: 2 red, 2 green and a blue one. The robot moves towards a box and the camera reads green. How likely is it that the box is actually green?

b) The robot's distance sensor can be modeled using a continuous random variable with a Normal distribution with $\sigma_1 = 0.3$ m. Express the sensor model $p(z|x)$ in the full form (not the shorthand notation).

c) Now the robot moves into another room that is empty. Initially it knows it is located at the door (x=0). The robot can execute *move* commands but the result of the action is not always perfect. Assume that the robot moves with constant speed v. The motion can also be modeled with a Gaussian with deviation $\sigma_2 = 0.1$ m. Write the motion model $p(x_t|x_{t-1}, u_t)$.

d) You let the robot run in the room with a speed of 1 m/s. The robot only runs forward and it updates its belief every second. Assume you get the following sensor measurements in the first 3 seconds: $\{z_1 = 1.2, z_2 = 1.6, z_3 = 2.5\}$.

Further assume that the position can only take discrete values from 0 to 5. Where does the robot believe it is located with respect to the door after 3 seconds? How certain is it about its location?

Exercise 6: An overview of ML methods

Try to find (for example by internet search or from the book (Bishop or)) at least 5 examples for learning techniques that have not been discussed in class. Describe these techniques briefly and classify them with respect to the categories presented in the lecture.

The next exercise class will take place on **April 29th, 2016**.

For downloads of slides and of homework assignments and for further information on the course see

`https://vision.in.tum.de/teaching/ss2016/mlcv16`
