

Weekly Exercises 8

Room: 02.09.023

Wed, 17.06.2015, 14:15-15:45

Submission deadline: Tue, 16.06.2015, 23:59 to windheus@in.tum.de

Please send in only Latex-PDF. If you have hand-written solutions, please hand them in during the lecture.

Functional maps

We will in the following denote by $\{\psi_i\}$ the standard basis (hat functions) and by $\{\phi_i\}$ the basis of Laplace Beltrami eigenfunctions.

Exercise 1 (One Point). One way to convert a functional map T_F to a point to point mapping is to apply T_F to special functions on the first shape, associated with the vertices (*indicator functions*) and then look for the indicator function on the second shape that is most similar (in the L^2 -sense) to the image.

1. Calculate the L^2 -norm of the function $f = \sum a_i \phi_i^M$ in terms of the coefficient-vector a .
2. Show that if we choose the indicator functions to be Diracs $\delta_j = M^{-1}e_j$ we can find the most similar indicator functions by comparing the columns of the matrix $C\Phi_M^T$ with the columns of Φ_N^T . Your calculations should start with

$$\|T_F(\delta_i^M) - \delta_j^N\|_{L^2(N)} = \dots$$

Programming: Functional Maps

Exercise 2 (Two points). Download and expand the file `ex8.zip` from the lecture website. Modify the files `spectralFM.m`, `fmToVertexMap.m` and `optimalFM.m` to implement the functions as explained below. You can run the script `exercise.m` to test and visualize your solutions.

`spectralFM.m` The function `C = spectralFM(P)` accepts a functional map `P` and returns an equivalent functional map `C`.

Functional map `P` is given as an n -by- n matrix and transforms the coordinates $\alpha \in \mathbb{R}^n$ of a real-valued function on mesh `M1` with respect to the basis of hat functions $\{\psi_i^{M1}\}_{1 \leq i \leq n}$ into the coordinates $\beta \in \mathbb{R}^n$ of a real-valued function on mesh `M2` with respect to the basis of hat functions $\{\psi_i^{M2}\}_{1 \leq i \leq n}$. I. e. let

$f = \sum_i \alpha_i \psi_i^{M1}$ be a function on $M1$ and let $g = \sum_i \beta_i \psi_i^{M2}$ the image of P at f , then $\beta = P\alpha$.

Functional map C should be an (almost) equivalent representation of P , where the functions are given as coordinates with respect to the eigen basis ($\{\phi_i^{M1}\}_{1 \leq i \leq k}$ and $\{\phi_i^{M2}\}_{1 \leq i \leq k}$) of the Laplace-Beltrami operator. I. e. C should be a k -by- k matrix.

What additional arguments will functions `spectralFM(...)` need? Please modify the script `exercise.m` accordingly.

`fmToVertexMap.m` The function `c = fmToVertexMap(C, Phi1, Phi2)` takes a functional map $C \in \mathbb{R}^{k \times k}$ between the two eigenspaces represented by matrices $\text{Phi1}, \text{Phi2} \in \mathbb{R}^{n \times k}$ and should return a point-to-point map represented by vector $\mathbf{c} \in \{1, \dots, n\}^n$ that assigns to each vertex on mesh 1 a corresponding vertex on mesh 2 by the following optimization problem:

$$c_i = \arg \min_j \|C \text{Phi1}_i^\top - \text{Phi2}_j^\top\|_2^2,$$

where Phi1_i^\top is the transposed of i -th row of matrix Phi1 and Phi2_j^\top is the transposed of j -th row of matrix Phi2 . Keep in mind that this point-to-point map is not necessarily injective.

Use matlab functions `KDTreeSearcher` and `knnsearch` to implement the function efficiently.

`optimalFM.m` Given some K vertex correspondences between two shapes the function `C = optimalFM(c, Phi1, M1, Phi2, M2)` should compute an optimal functional map $C \in \mathbb{R}^{k \times k}$ in the least-squares sense as described in the lecture. The correspondences are given by matrix $\mathbf{c} \in \{1, \dots, n\}^{K \times 2}$, such that the i -th correspondence associates the $c_{i,1}$ -th vertex on shape 1 to the $c_{i,2}$ -th vertex on shape 2. $\text{Phi1}, M1, \text{Phi2}, M2$ are the eigenbasis and mass matrices for shape 1 and shape 2.

You can assume $K \geq k$. How can you construct vectors $a_i, b_i \in \mathbb{R}^k$ from the given correspondences?