Analysis of Three-Dimensional Shapes
E. Rodolà, T. Windheuser, M. Vestner
Summer Semester 2015

Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 7

Room: 02.09.023
Wed, 10.06.2015, 14:15-15:45
Submission deadline: Tue, 09.06.2015, 23:59 to windheus@in.tum.de
Please send in only Latex-PDF. If you have hand-written solutions, please hand them in during the lecture.

Heat Kernel

Recall the definition of the *heat kernel* from the lecture. Given a shape $\mathcal{S}$ and time parameter $t \in \mathbb{R}_{\geq 0}$, the heat kernel k_t is defined by

$$k_t(x, y) = \sum_i \exp(-\lambda_i t)\phi_i(x)\phi_i(y)$$

for any two points $x, y \in \mathcal{S}$, where $\lambda_i \in \mathbb{R}_{\geq 0}, \phi_i : \mathcal{S} \to \mathbb{R}$ are the eigenvalues and eigenfunctions of the Laplace-Beltrami operator, respectively.

Exercise 1 (One Point). Show that the heat kernel satisfies the properties of a *diffusion kernel*:

1. $\forall x, y \in \mathcal{S} : k_t(x, y) \geq 0$ (non-negativity),
2. $\forall x, y \in \mathcal{S} : k_t(x, y) = k_t(x, y)$ (symmetry),
3. $\int_\mathcal{S} \int_\mathcal{S} (k_t(x, y))^2 dxdy < \infty$ (square integrability),
4. $\int_\mathcal{S} \int_\mathcal{S} k_t(x, y) f(x) f(y) dxdy \geq 0$ for any $f : \mathcal{S} \to \mathbb{R}$ (positive semi-definiteness),
5. $\int_\mathcal{S} k_t(x, y) dy = 1$ for any $x \in \mathcal{S}$ (positive semi-definiteness).