# Multiple View Geometry: Examination Preparation

Exercise: 9 August 2011

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Critical surfaces: To have a unique solution (up to a scalar factor), it is very important for the points considered in the above six-point or eight-point algorithms to be in general position. If a (dense) set of points whose images allow at least two distinct essential matrices, we say that they are "critical". Let $X \in \mathbb{R}^3$ be coordinates of such a point and $(R, T)$ be the motion of a camera. Let $x_1 \sim X$ and $x_2 \sim (RX + T)$ be two images of the point.

   (a) Show that if
   $$(RX+T)^T \hat{T}' R' X = 0,$$
   then
   $$x_2^T \hat{T} R x_1 = 0, \quad x_2^T \hat{T}' R' x_1 = 0.$$

   (b) Show that for points $X$ that satisfy the equation $(RX+T)^T \hat{T}' R' X = 0$, their homogeneous coordinates $\bar{X} = [X, 1]^T \in \mathbb{R}^4$ satisfy the quadratic equation
   $$\bar{X}^T \begin{bmatrix} R^T \hat{T}' R + R'^T \hat{T'}^T R & R'^T \hat{T'}^T T \\ T^T \hat{T}' R' & 0 \end{bmatrix} \bar{X} = 0.$$
   This quadratic equation is called a *critical surface*. So no matter how many points one chooses on such a surface, their two corresponding images always satisfy epipolar constraints for at least two different essential matrices.

2. Consider the matrix $R = I - 2nn^\top$ with the $3 \times 3$ identity matrix $I$ and vector $n \in \mathbb{R}^3$ with unit length, i.e. $|n| = 1$.

   (a) Show that $R^{-1} = R^\top = R$.

   (b) Show that one of the eigenvalues of $R$ is $-1$ and two eigenvalues are 1. What does this mean for the corresponding eigenvectors?

   (c) Show that $R \in O(3)$ and $\det(R) = -1$, i.e. $R$ is a reflection matrix.

3. Given a reflective symmetry $g = [R, 0]$ with the reflection matrix $R$ and camera pose $[R_0, T_0]$, the relative pose between a given image and an equivalent one is $R' = R_0 R R_0^\top$ and $T' = T_0 - R' T_0$.

   (a) Show that $R'T' = -T'$ and $R'\hat{T}' = \hat{T}' R = \hat{T}'$.

   (b) Show that $R'$ performs a reflection about the plane that is perpendicular to $T'$, i.e. $R' = I - 2nn^\top$ with normal $n = T'/|T'|$.

   *Hint: Consider the following property of skew-symmetric matrices: $\widehat{Au} = A\hat{u}A^\top$ for $u \in \mathbb{R}^3$ and $A \in SO(3)$.*