# Multiple View Geometry: Exercise Sheet 6

Exercise: 5 July 2011

## Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $F \in \mathbb{R}^{3\times3}$ the fundamental matrix for cameras $C_1$ and $C_2$. Show that the following holds for the epipoles $e_1$ and $e_2$:

$$Fe_1 = 0 \qquad\qquad e_2^T F = 0$$

2. The essential matrix $E := \hat{T}R$ with $T \in \mathbb{R}^3$ and $R \in \mathrm{SO}(3)$ has the singular value decomposition $E = U\Sigma V^T$. Let further $R_Z\left(\pm\frac{\pi}{2}\right)$ be the rotation by $\pm\frac{\pi}{2}$ around the $z$-axis. Show the following properties:

   (a) $\hat{T} = UR_Z\left(\pm\frac{\pi}{2}\right)\Sigma U^T \in so(3)$, i.e. is a skew-symmetric matrix.

   (b) $R = UR_Z\left(\pm\frac{\pi}{2}\right)^T V^T \in \mathrm{SO}(3)$, i.e. is a rotation matrix.

3. Consider matrices $E = \hat{T}R$ and $H = R + Tu^T$ with $R \in \mathbb{R}^{3\times3}$ and $T, u \in \mathbb{R}^3$. Show that the following holds.

   (a) $E = \hat{T}H$

   (b) $H^T E + E^T H = 0$

## Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Download the package `mvg_ex06.tgz` from the website and extract the images `batinria0.tif` and `batinria1.tif`.

2. Show the first image and mark at least 8 points. You can retrieve the pixel coordinates of mouse clicks with the command `[x,y] = ginput(gcf)`. Then show the second image and click at the corresponding points in the same order. Again you can get the pixel coordinates with `ginput`. Now you should have the 2D coordinates of corresponding point pairs.

3. Implement the 8-point algorithm from the lecture and run it with these point pairs. To this end, you have to transform the coordinates. The intrinsic camera matrices are:

$$K1 = \begin{pmatrix} 844.310547 & 0 & 243.413315 \\ 0 & 1202.508301 & 281.529236 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K2 = \begin{pmatrix} 852.721008 & 0 & 252.021805 \\ 0 & 1215.657349 & 288.587189 \\ 0 & 0 & 1 \end{pmatrix}$$

Hints:

- kron
- reshape
- It might occur that one of the matrices $U$ oder $V^\top$ of the SVD of $E$ has a determinant of $-1$. In this case, determine the SVD of $-E$.

4. Reconstruct the depths of the points as described on slides 16 and 17.

**Matlab-Tutorials:**
http://www.math.utah.edu/lab/ms/matlab/matlab.html
http://www.math.ufl.edu/help/matlab-tutorial/
http://www.glue.umd.edu/˜nsw/ench250/matlab.htm