Multiple View Geometry: Exercise Sheet 4 & 5

Exercise: 21 June 2011

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

Part 1: Image Formation

1. A classic ambiguity of the perspective projection is that one can not tell an object from another object that is exactly *twice as big but twice as far*. Explain why this is true.

2. Not only the projection, but all series of translations and rotations can be expressed as matrix multiplications in homogeneous coordinates (as shown on the last exercise sheet). Give the projection matrix P_{RT} for a pinhole camera, that is first rotated with $R \in SO(3)$ and the focus of which is then translated by F.

3. Consider a 3D reconstruction of a single point $X = (0\ 0\ 4)^\top$. The point is observed by two cameras given by the following projection matrices:

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad\qquad P_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

   (a) Compute the images x and y of the point X after projection for camera 1 and 2, respectively.

   (b) Compute the resulting discretized image coordinates $\hat{x}$ and $\hat{y}$ (round to integer numbers). Calculate the corresponding preimages of $\hat{x}$ and $\hat{y}$ and check whether they intersect.

Part 2: The Lucas-Kanade method

1. For which translational velocity is the energy of the approach of Lucas and Kanade (see slide 8) minimized? (Consider the region as given)

2. Compute the corresponding formula for affine motion.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

Part 1: Image Formation

1. Consider the 3D model `model.off` from the last exercise contained in the package `mvg_ex02.tgz` and a camera centered at $C = (0, 0, -1)^T$ with focal length $f = 1$.

   (a) Compute the perspective projection of the model using a homogenuous projection matrix. To this end, you need to transform the list of vertices returned by `openOFF` into homogeneous coordinates.

   (b) Consider a parallel projection where the projection rays are parallel to the z-axis. What is the corresponding projection matrix? Use this matrix to project the model onto the image plane.

   (c) Compute a spherical projection of the model onto the unit sphere $\mathbb{S}^2$.

Part 2: The Lucas-Kanade method

1. Download the package `mvg_ex05.tgz` from the website and extract the images `street1.pgm` and `street2.pgm`. Read the images with the matlab command `imread` and visualize them using `imshow` or `imagesc`.

2. Compute the image gradients I_x, I_y and I_t with the following discretization schemes:

$$I_x = \frac{I(x+1,y) - I(x-1,y)}{2}, \;\; I_y = \frac{I(x,y+1) - I(x,y-1)}{2}, \;\; I_t = I_2 - I_1$$

3. Compute the structure tensor at each pixel.

4. Compute the velocity for all pixels for which the determinant of the structure tensor is larger than a threshold θ. To this end make use of the formula of Lucas and Kanade θ (see slide 10).

Matlab-Tutorials:
`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`
`http://www.glue.umd.edu/~nsw/ench250/matlab.htm`