Multiple View Geometry: Exercise Sheet 3

Exercise: 7 June 2011

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $R \in \mathrm{SO}(3)$ be a diagonalizable 3×3-matrix with eigenvalues $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$.
   (a) Show that $|\lambda_i| = 1$ holds for all eigenvalues λ_i.
   (b) Show that for eigenvalue λ its corresponding complex conjugate $\bar{\lambda}$ is also an eigenvalue.
   (c) Show that $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1$ holds.
   (d) Show that one of the eigenvalues is 1. What does this mean for its corresponding eigenvector?

   Hint: *Consider the characteristic polynomial of R, i.e.:* $t \mapsto \det(R - t\,\mathrm{Id})$.

2. Consider a vector $\omega \in \mathbb{R}^3$ with $||\omega|| = 1$ and its corresponding skew-symmetric matrix $\hat{\omega}$.
   (a) Show that $\hat{\omega}^2 = \omega\omega^\top - \mathrm{Id}$ and $\hat{\omega}^3 = -\omega$.
   (b) Following the result of (a), find simple rules for the calculation of $\hat{\omega}^n$. Distinguish between odd and even numbers n.
   (c) Derive Rodriguez' formula:

$$e^{\hat{\omega}} = I + \frac{\hat{\omega}}{||\omega||}\sin(||\omega||) + \frac{\hat{\omega}^2}{||\omega||^2}(1 - \cos(||\omega||))$$

3. Consider a continuous family of rigid body transformations:

$$g(t) = \begin{pmatrix} R(t) & T(t) \\ 0 & 1 \end{pmatrix} \in \mathbb{R}^{4\times 4}$$

   (a) Calculate $\dot{g}(t)$ and $g^{-1}(t)$
   (b) Calculate the twist $\dot{g}(t) \cdot g^{-1}(t)$

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. (a) Write a function which takes a vector $w \in \mathbb{R}^3$ and returns its corresponding element $R = e^{\hat{w}} \in SO(3) \subset \mathbb{R}^{3\times 3}$ from the Lie group. Hence, the function will be a concatenation of the hat operator $\wedge$ and the exponential mapping.
   (b) Implement another function which performs the corresponding inverse transformation and test the two functions on some examples.
   (c) Implement similar functions which calculate the transformation for twists, i.e. from $\xi \in \mathbb{R}^6$ to $e^{\hat{\xi}} \in SE(3) \subset \mathbb{R}^{4x4}$ and the other way around.

2. Let $R \in SO(3)$ be a rotation matrix generated by rotation about a unit vector ω by θ radians that satisfies $R = \exp(\hat{\omega}\theta)$. Suppose R is given as

$$R = \begin{pmatrix} 0.1729 & -0.1468 & 0.9739 \\ 0.9739 & 0.1729 & -0.1468 \\ -0.1468 & 0.9739 & 0.1729 \end{pmatrix} \tag{1}$$

   (a) Use the formula given in the lecture to compute the rotation axis and the associated angle.

   (b) Use Matlab's function `eig` to compute the eigenvalues and eigenvectors of rotation matrix R. What is the eigenvector associated with the unit eigenvalue? Give its form and explain its meaning.

3. In exercise 2 you have already implemented a function which rotates a model around a given point. Extend this code by a function `W = rotate_around_ray(V,a,b,angle)` which rotates a model - given as vertex list (matrix) `V` - around a ray defined by the offset vector `a` and direction vector `b`. The function output is the transformed vertex list `W`.
   Make use of Rodrigues' formular and check whether the created matrix is orthogonal. Test your method with the model given in `model.off`. For instance, rotate the model by 50 degrees around the ray through point `a = (0.5 0 0.45)`^{T} and direction `b = (0 1 0)`^{T}.

Matlab-Tutorials:

`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`
`http://www.glue.umd.edu/~nsw/ench250/matlab.htm`