An Adaptive Confidence Measure for Optical Flows Based on Linear Subspace Projections

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Abstract. Confidence measures are important for the validation of optical flow fields by estimating the correctness of each displacement vector. There are several frequently used confidence measures, which have been found of at best intermediate quality. Hence, we propose a new confidence measure based on linear subspace projections. The results are compared to the best previously proposed confidence measures with respect to an optimal confidence. Using the proposed measure we are able to improve previous results by up to 31%.

1 Introduction

Optical flow calculation is a crucial step for a wide variety of applications ranging from scientific data analysis and medical imaging to autonomous vehicle control and video compression. Methods for optical flow computation can be distinguished into local and global approaches. Most local approaches are either based on the idea of Lucas and Kanade [7], Bigün [11] or on the method of Anandan [2], where an energy term is minimized for each pixel individually under utilization of a small neighborhood. Global techniques usually follow the concept of Horn and Schunck [8], which implements prior knowledge on the flow field by spatio-temporally relating neighboring flow estimates by means of global energy functionals. One of the most accurate methods recently has been proposed by Bruhn et al. [9] and combines the advantages of both approaches.
Confidence measures are indispensable to assess and improve the quality of optical flow fields. In 1994, in their landmark paper Barron and Fleet [3] stated that "confidence measures are rarely addressed in literature" even though "they are crucial to the successful use of all [optical flow] techniques". Using the information provided by confidence measures, the accuracy of the estimated flow field can be improved by integrating the confidence measure into the calculation method or by postprocessing, e.g. removing and reconstructing incorrect flow vectors. We have to distinguish between confidence and situation measures. Despite different concepts, both types of measures have been used as confidence

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measures in literature. Situation measures assess the mere possibility of an accurate flow computation and yield a low value for example in cases of occlusions, transparent structures, severe noise, aperture problems and homogeneous regions. However, the actual displacement vector is not necessarily considered and can be assessed as unreliable despite its correctness. In contrast, confidence measures evaluate the correctness of a given flow vector independent of the situation in the sequence. A comparison of known situation and confidence measures can be found in [1]. So far, for variational methods only one general type of explicit confidence measures has been proposed in [10]. The idea is to use the inverse of the global energy functional in order to detect violations of the flow computation model. For the well-known structure tensor method proposed by [11] the only confidence measures that have been proposed actually belong to the class of situation measures, as they do not take into account the displacement vector at all when assessing its correctness, e.g. the gradient of the image or other measures described in [4, 13, 12]. Furthermore, the quality of these measures is unsatisfactory as we have pointed out in [1]. We have also proposed several variational data terms such as the brightness constancy equation and various regularizers [6, 5] as confidence measures for the structure tensor method. In fact, these measures turned out more reliable than situation measures, but still did not obtain accurate results. This is mainly due to the problem that each measure assumes a specific flow computation model, which cannot be valid in any situation. Therefore, we propose a new confidence measure for the structure tensor method that is adaptable to the current flow computation problem by means of unsupervised learning. In fact, the measure can be used for all optical flow fields that have been computed with no or minor smoothness assumptions. Even ground truth data which is generally unavailable is not necessary as the model can be learned either from a set of ground truth flow fields or from a previously computed flow field. The linear subspace projection method has been applied for the estimation of optical flows before, directly by Black et al. in [17] and by means of Markov Random Fields by Roth and Black in [16]. In contrast to these approaches, where only spatial information is used, we extend the subspace method to include temporal information of the flow field and derive a new confidence measure. The concept of our confidence measure is based on the idea of learning typical displacement vector constellations within a local neighborhood. The resulting model consists of a set of basis flows, a linear subspace of the flow field, that is sufficient to reconstruct 99% of the information contained in the flow field. Displacement vectors that cannot be reconstructed by this model are considered unreliable. Hence, the reconstruction error is chosen as confidence measure. It performs better than previously proposed confidence measures and obtains a substantial gain of quality in several cases.

2 Definitions

Let a given image sequence $\mathcal{I}$ be defined on a time interval $[0, T]$ as

$$\mathcal{I} : \Omega \times [0, T] \to \mathbb{R},\ \Omega \subseteq \mathbb{R}^2 \tag{1}$$

The notion "optical flow" refers to the displacement field u of corresponding pixels in subsequent frames of an image sequence

$$u : \Omega \times [0, T] \rightarrow \mathbb{R}^2 \tag{2}$$

Then a confidence measure is a mapping c from the image sequence and a two-dimensional displacement vector to the interval [0..1]:

$$c : \mathcal{I} \times u \rightarrow [0..1] \tag{3}$$

There is an infinite number of possible displacement vector constellations. Hence, none of the previously proposed models is able to represent all of them. Using given ground truth flow fields or computed flow fields we try to derive as much information as possible from the samples in order to incorporate this information into the learned flow model. Since much of the information contained in a flow field is only obvious in the temporal domain, the inclusion of temporal information is indispensable. However, it is rarely used in literature.

3 Linear Subspace Projections

In order to learn the linear subspace model any unsupervised learning method can be used. We apply principal component analysis (PCA) and use a set of given displacement fields for the training. They consist of a horizontal and a vertical flow component. To compute the principal components a spatially and temporally distributed neighborhood containing both components is read into a single vector using a lexicographic order. The matrix containing a large number of such sample vectors will be called M. The idea of PCA is to find a low-dimensional subspace which preserves as much information (variance) of the dataset as possible and in which the different dimensions of the data are decorrelated. The covariance of the matrix M represents the correlation of the dataset along each two dimensions. Hence, the goal is to find a new basis system, where the covariance of each two dimensions is zero, that means the covariance matrix of M is diagonal. As the covariance matrix of M is symmetric and positive definite it can be diagonalized. We can obtain such a basis system by finding an orthogonal matrix S and applying the similarity transformation

$$D = S^T \; Cov(M) \; S \tag{4}$$

e.g. using Givens rotations. The matrix S then contains the eigenvectors of the covariance matrix of M, which form the new basis system. In order to reduce the dimensionality of the data to a meaningful subspace, the axes representing the least information (the smallest variance) of the dataset can be removed. These are the eigenvectors with the smallest eigenvalues. We can select the number of eigenvectors containing the fraction δ of the information of the original dataset by choosing k of the n eigenvectors v_i sorted by decreasing eigenvalue such that

$$\frac{\sum_{i=1}^{k} v_i}{\sum_{i=1}^{n} v_i} \geq \delta \tag{5}$$

With the eigenvectors (*"eigenflows"*) we can now approximately reconstruct any displacement vector neighborhood $\mathcal{N}_{\boldsymbol{x}}$ centered on position $\boldsymbol{x}$ by a linear combination of the k selected eigenflows using the reconstruction function r

$$r(\mathcal{N}_{\boldsymbol{x}}, k) = \sum_{i=1}^{k} \alpha_i v_i + m \ , \ m = \frac{1}{n} \sum_{i=1}^{n} s_i \tag{6}$$

where s_i are the data samples in the columns of M. In order to obtain the coefficient vector α containing the eigenflow coefficients α_i, it is sufficient to project the sample neighborhood $\mathcal{N}_{\boldsymbol{x}}$ into the linear subspace spanned by the eigenflows using the transformation

$$\alpha = S^T(\mathcal{N}_{\boldsymbol{x}} - m) \tag{7}$$

Figure 1 shows examples for eigenflows derived from computed flow fields. Using temporal information the resulting eigenflows can represent complex temporal phenomena such as a direction change, a moving motion discontinuity or a moving divergence.

The linear combinations of the previously derived eigenflow vectors represent typical flow field neighborhood constellations. Depending on the training data the information contained in the learned model varies. If ground truth flow fields are used many sample sequences are necessary to include most of the possible flow constellations. However, as only very few sequences with ground truth exist the resulting eigenflows only represent an incomplete number of constellations. In contrast, it is possible to compute the flow for a given sequence and use exactly this computed flow as input for the unsupervised learning algorithm. In this way the resulting model will be well adapted to the current flow problem. However, if the flow computation method does not allow certain displacement vector constellations the trained linear subspace will not be sufficient to represent these constellations either, as all training samples are derived from the computed flow field. This is for example the case if the flow computation model demands a smooth flow field, which leads to the problem that the sample flows do not contain any flow edges. In both cases, if we learn from insufficient ground truth flows or from incorrect, computed flow fields, the problem that correct flow constellations cannot be reconstructed from the eigenflows persists. We will compare both methods.

4 A Confidence Measure from Eigenflows

To evaluate the confidence of a given flow vector we have to consider its validity within its spatio-temporal context, that is within its neighborhood $\mathcal{N}_{\boldsymbol{x}}$ of flow vectors. Given a number of k model parameters, e.g. eigenflows, a confidence measure can be derived based on the assumption that displacement vectors are the more reliable within their neighborhood the better they can be reconstructed from the eigenflows, which represent typical flow constellations. Hence, the reconstruction error of the flow vector will serve as confidence measure:

$$c(\boldsymbol{x}, \boldsymbol{u}) = \varphi(\boldsymbol{u}, r(\mathcal{N}_{\boldsymbol{x}}, k)) \tag{8}$$

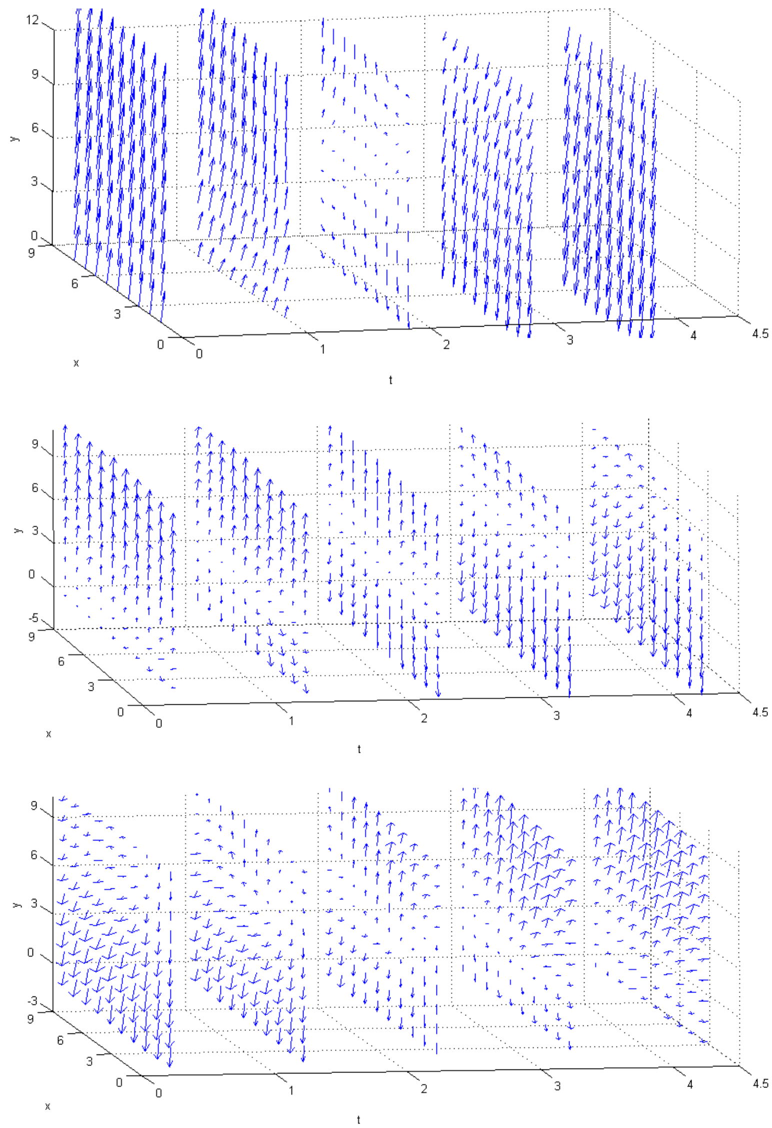

Fig. 1. Examples for eigenflows calculated from computed flow fields using spatial and temporal information; the inclusion of temporal information allows the representation of complex temporal phenomena such as a flow direction change (top), a moving motion discontinuity (center) and a moving divergence (bottom)

The size of the neighborhood $\mathcal{N}_{\boldsymbol{x}}$ of course has to be the same as for the eigenflows. The function φ represents the error measure evaluating the similarity between the calculated flow vector $\boldsymbol{u}$ and the reconstructed vector $r(\mathcal{N}_{\boldsymbol{x}}, k)$.
The correctness of computed flow fields is usually evaluated in terms of the angular error. Thus, we will use this error measure to define the function φ. Let the displacement vectors $\boldsymbol{u} = (u_1, u_2)$ be represented as 3-dimensional vectors of unity length $\overline{u} = \frac{1}{\sqrt{u_1^2+u_2^2+1}}(u_1, u_2, 1)$. Then we can derive the function φ based on the angular error α:

$$\varphi(\boldsymbol{u}, \boldsymbol{v}) = 1 - \alpha(\boldsymbol{u}, \boldsymbol{v}) \ , \ \alpha(\boldsymbol{u}, \boldsymbol{v}) = \frac{arccos(\overline{u} \cdot \overline{v})}{\pi} \tag{9}$$

The new confidence measure will be called pcaReconstruction measure.
Our proposed method may fail on rare occasions of untypical flows encountered in the imaged data. These are singular events which in case of underrepresentation in the training data may not be adequately incorporated into our basic PCA framework. A range of more refined algorithms have been developed in the field of statistical learning. Some of these might solve this problem, such as multiclass PCA [18] or partial least squares regression.

5 Evaluation of Confidence Measures

For the comparison of the proposed confidence measures to the best previously proposed confidence measures, we use the technique presented in [1]. It is based on the gradual sparsification (successive removal of least reliable flow vectors to reduce the computation error) of the flow fields. The following problems, which make a fair comparison very difficult, have been stated and solved in [1].

1. The confidence measures are bounded by the interval [0,1], but they all follow different scales, which do not necessarily span all possible values. Therefore a comparison of absolute confidence values is impossible.
2. The confidence values are often highly non-linear.

The basic idea for the comparison of a given to the optimal confidence map is to compare the sparsification order of both confidence measures. In this way nonlinearities and different scales do not influence the result. Thus, ground truth for a given confidence measure is necessary. It is defined using the angular error of the computed flow vector $\boldsymbol{u}(\boldsymbol{x})$ and the ground truth flow vector $\boldsymbol{g}(\boldsymbol{x})$

$$c_{opt}(\boldsymbol{x}) = \varphi(\boldsymbol{u}(\boldsymbol{x}), \boldsymbol{g}(\boldsymbol{x})) \tag{10}$$

In [1] we have proposed the confidence measure quality value (CMQV) for the evaluation of a given confidence measure. Let Q be the set of all pixels q in the image sequence. The intuition is to "punish" any deviation from the optimal sparsification *order*. This punishment $P(q)$ at pixel q is weighted by the "damage" $D(q)$ this error in the sparsification order causes. $D(q)$ is defined as the difference in the error that could have been removed in this situation and the error that has been removed by eliminating the current flow vector. Finally the quality value of the confidence measure is calculated as the average over all punishment values weighted by the damage values for all pixels in the image sequence.

$$CMQV(Q) := \frac{\sum_{q \in Q} P(q)D(q)}{|Q|} \quad (11)$$

6 Results

The test of the proposed confidence measure is carried out as described in [1]. We use four test sequences to compare our results to previously known confidence measures based on the CMQV values defined in 11: the Marble and Yosemite sequences as well as the Street and the Office sequences [14] shown in Figure 2.

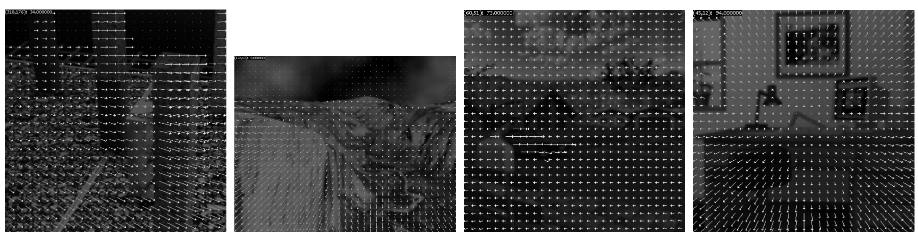

Fig. 2. A sample frame of each Marble, Yosemite, Street and Office sequence

We use the CMQV values as basis for the comparison of the proposed confidence measure and the best known measures from literature. A flow field calculated by the structure tensor method with the same parameters as in [1] (7x7x7 flow optimal derivative filters of Scharr [15], structure tensor integration scale σ=4) is used for the confidence measure test. As explained above, for the computation of eigenflows ground truth or computed flow fields can be used. Table 1 shows the best experimentally determined parameters (number n and spatio-temporal size (x,y,t) of eigenflows in the format (n,x=y,t)) for the pcaReconstruction measure for each sequence based on eigenflows computed from ground truth and calculated flow fields.

	Marble	Yosemite	Street	Office
ground truth	2.01 (2, 11, 5)	1.81 (6, 3, 1)	1.64 (4, 9, 3)	2.35 (14, 3, 3)
computed	1.98 (5, 19, 5)	1.72 (5, 3, 1)	1.60 (4, 9, 5)	2.38 (5, 3, 3)

Table 1. CMQV values scaled by a factor of 100 for the pcaReconstruction measure on all four test sequences for eigenflows based on ground truth flow fields (top) and on computed flow fields (bottom); parameters: number n and spatio-temporal size (x,y,t) of eigenflows in the format (n, x=y, t)

The results show that the difference between the eigenflows learned from ground truth and from computed flow fields is almost negligible. The results are even slightly better for computed flow fields in three of the four sequences. Hence, we can conclude that ground truth flow fields are not absolutely necessary for the successful application of the proposed confidence measure. Table 2 contains the ten best confidence measures for the four sequences. The best previously proposed confidence measures are based on energy terms (similarity terms or regularizers) of global flow computation methods. Most of them are derived from [6, 5, 4] and are assembled and compared in [1].

Marble	Yosemite	Street	Office
anisoFlowReg (1.85)	**pcaReconst (1.72)**	isoFlowReg (1.44)	**pcaReconst (2.38)**
isoImReg (1.86)	structCt (2.49)	anisoFlowReg (1.47)	spaceTimeReg (3.45)
isoFlowReg (1.86)	isoImReg (2.96)	**pcaReconst (1.60)**	timeReg (3.49)
tvReg (1.88)	crossCorr (3.02)	tvReg (1.61)	anisoImReg (3.50)
homReg (1.88)	anisoImReg (3.06)	anisoImReg (1.64)	isoImReg (3.50)
anisoImReg (1.93)	structEv3 (3.06)	isoImReg (1.78)	tvReg (3.50)
pcaReconst (1.98)	ssd (3.06)	homReg (1.78)	anisoFlowReg (3.51)
timeReg (2.02)	anisoFlowReg (3.08)	spaceTimeReg (1.93)	homReg (3.56)
spaceTimeReg (2.23)	isoFlowReg (3.09)	structEv3 (1.96)	isoFlowReg (3.60)
laplaceConst (2.55)	homReg (3.20)	hessConst (2.13)	structCt (3.94)

Table 2. Comparison of new confidence measure (pcaReconst) to 9 best known confidence measures; the CMQV values are scaled by a factor of 100

The results show that the proposed pcaReconstruction confidence measure always ranges among the seven best (out of 33) known confidence measures. For the Marble and the Street sequence the results are comparable to those of the best known confidence measure. In contrast to that, the results on the Yosemite and the Office sequence are far superior to those of the best known confidence measure as the CMQV values could be reduced by 31%. This corresponds to an average reduction of the CMQV value by 11.8 %. The resulting confidence maps compared to the optimal confidence are depicted in Figure 3.

7 Summary and Conclusion

We have presented a new confidence measure based on linear subspace projections using unsupervised learning. It can be used in combination with any flow computation method not demanding strong smoothness constraints, e.g. the structure tensor method. Ground truth sequences, which are usually unavailable, are not necessary to obtain high quality results - instead the performance is slightly superior for models learned from computed flow fields. Tests indicate that for the chosen test sequences the new measure significantly outperforms previously proposed measures.

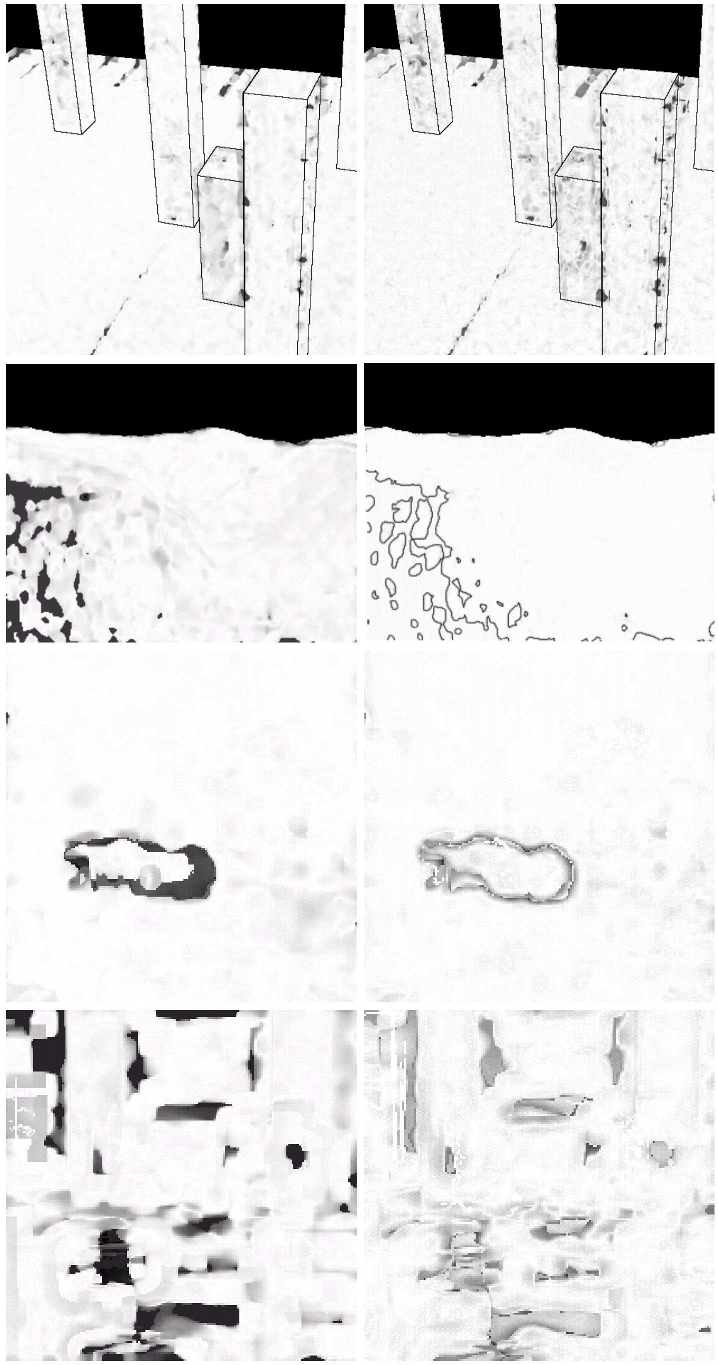

Fig. 3. Comparison to optimal confidence, left: optimal confidence map, right: pcaReconstruction confidence map

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