TU München
Fakultät für Informatik
PD Dr. Rudolph Triebel
John Chiotellis

Machine Learning for Robotics and Computer Vision
Summer term 2016

Homework Assignment 5

Topic 1: Kernels
June 13, 2016

Exercise 1: Constructing kernels

Let k_1 and k_2 be kernels, $f : \mathbb{R}^n \to \mathbb{R}$ an arbitrary function . Show that we can construct new kernels via

a) $k(x_1, x_2) = k_1(x_1, x_2) + k_2(x_1, x_2)$

b) $k(x_1, x_2) = k_1(x_1, x_2)k_2(x_1, x_2)$

c) $k(x_1, x_2) = f(x_1)k_1(x_1, x_2)f(x_2)$

d) $k(x_1, x_2) = \exp(k_1(x_1, x_2))$

e) $k(x_1, x_2) = x_1^T A x_2$, where A symmetric, positive semi-definite $n \times n$ matrix

Exercise 2: Polynomial kernel

Let $x_i, x_j \in \mathbb{R}^2$

a) Show (by induction) that $k_d(x_i, x_j) = (x_i^T x_j)^d$ is a kernel for every $d \geq 1$.

b) Find $\phi_d(x)$ such that $k_d(x_i, x_j) = \phi_d(x_i)^T \phi_d(x_j)$.

c) Find $\tilde{\phi}_2(x)$ for $\tilde{k}_2(x_i, x_j) = (x_i^T x_j + d)^2$ $(d > 0)$.

Exercise 3: Feature Spaces

Consider a dataset with a single feature $x \in \mathbb{R}$ and labels $y \in \{+1, -1\}$. Data points $-3, -2, 3$ have label $+1$ and data points $-1, 0, 1$ have label -1.

a) Is this dataset linearly separable? Why?

b) Find a feature map $\phi(x) \in \mathbb{R}^2$ so that the dataset is linearly separable.
(Drawing the data helps.)

c) Considering the determinant of a 2×2 Gram matrix show that a positive definite kernel satisfies the Cauchy-Schwartz inequality.

Topic 2: Gaussian Processes

Exercise 4: Gaussian Processes Regression

Consider a GP regression model in which the kernel function is defined in terms of a fixed set of nonlinear basis functions. Show that the predictive distribution is identical to the one of the Bayesian linear regression model (see Lecture and Homework Assignment 2).

Hint 1: Both models have Gaussian predictive distributions.
Hint 2: Make use of:

$$(I + AB)^{-1}A = A(I + BA)^{-1}$$

and the Woodburry identity:

$$(A + BD^{-1}C)^{-1} = A^{-1} - A^{-1}B(D + CA^{-1}B)^{-1}CA^{-1}$$

Exercise 5: Gaussian Processes Classification : Programming

Visit `http://gaussianprocess.org/`. You will find a vast amount of resources relevant to Gaussian processes, including research papers and software. For this exercise we will use the *gpml* package for Matlab (written by Rasmussen and Williams). Read through the documentation in `http://www.gaussianprocess.org/gpml/code/matlab/doc/index.html`. Experiment with gpml for classification using the dataset from the previous exercise (banknote_auth). Section 4(b) of the documentation can guide you through the different parameters you can tinker with, like mean, covariance and likelihood functions. How do GPs compare to AdaBoost?

The next exercise class will take place on **June 24th, 2016**.

For downloads of slides and of homework assignments and for further information on the course see

`https://vision.in.tum.de/teaching/ss2016/mlcv16`
